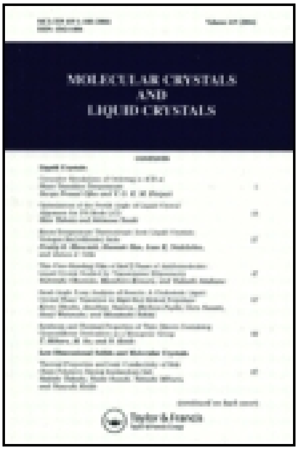

Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl20

Theoretical Analyses of the Electric Field Control of Focal Length in a Gradient Polymer Stabilized Liquid Crystal Lens

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Version of record first published: 22 Sep 2006

To cite this article: Victor Yu. Reshetnyak, Svitlana L. Subota & Tigran V. Galstian (2006): Theoretical Analyses of the Electric Field Control of Focal Length in a Gradient Polymer Stabilized Liquid Crystal Lens, Molecular Crystals and Liquid Crystals, 454:1, 187/[589]-200/[602]

To link to this article: http://dx.doi.org/10.1080/15421400600654090

Mol. Cryst. Liq. Cryst., Vol. 454, pp. 187/[589]–200/[602], 2006
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ISSN: 1542-1406 print/1563-5287 online
DOI: 10.1080/15421400600654090

Theoretical Analyses of the Electric Field Control of Focal Length in a Gradient Polymer Stabilized Liquid Crystal Lens

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In this paper we study the process of network formation, director reorientation upon applied voltage and electro-optical properties of the gradient polymer stabilized liquid crystal (G-PSLC) lens.

Using classical scheme of radical photopolymerization the profile of the polymer network concentration is obtained. Minimizing the total free energy functional we find the director profile in the cell subject to externally applied electric field. We suppose that polymer network creates additional torque onto LC director forcing it to preserve planar orientation competing with applied voltage. Thus, inhomogeneous director reorientation profile arises despite the use of uniform electrodes. As a result inhomogeneous refractive index is obtained, which generates the lens with focal length dependant from applied voltage.

Obtained results can be applied to develop G-PSLC lenses that have no moving parts and allow the electro-optical zooming.

Keywords: Freedericksz transition; polymer stabilized liquid crystals; tunable lens

The authors would like to acknowledge the support of Natural Science and Engineering Research Council NSERC of Canada and Photintech Inc. for their financial support of this work.

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1. INTRODUCTION

Active optical elements, such as liquid crystal (LC) adaptive lenses, are interesting for many applications in intelligent (self-adjusting) vision systems thanks to their compactness and easy driving. Most frequently, a non-homogeneous electric field is applied to flat LC cell to create lens-like distribution of the refractive index via inducing suitable LC orientational structure [1–7]. Homogeneous electric field may be used also in combination with micro-patterned LC orientation to produce a Fresnel diffraction lens [8] or with surface relief micro lenses immersed in the LC cell [9]. Recently a new polymer stabilized LC structure has been proposed for a tunable lens application [10]. The mixture of homogeneous planar oriented nematic liquid crystal (NLC) and few percent of photopolymerizable monomer are illuminated by a laser beam with Gaussian spatial intensity distribution, $I(\rho) = I_0 \exp\left(-\alpha\rho^2\right)$. This exposition induces a spatially inhomogeneous polymer network [10,11] in the cell (Fig. 1).

As a result, the electro-optical response of this system to a uniform electric field reveals a nonhomogeneous, centrally symmetric, character (Fig. 1). Director deviation appears to be minimal in the centre of the illuminated spot and maximal in the nonilluminated regions. Such cell represents a positive lens for the extraordinary polarized light. By applying the voltage one can vary the profile of the refractive index and so, the focal length of the lens.

The present work provides theoretical analyses of operation of the above mentioned system. The paper is organized as follows. In the first Section we find the spatial profile of polymer network and its relation to the spatial distribution of the illuminating UV light. The second

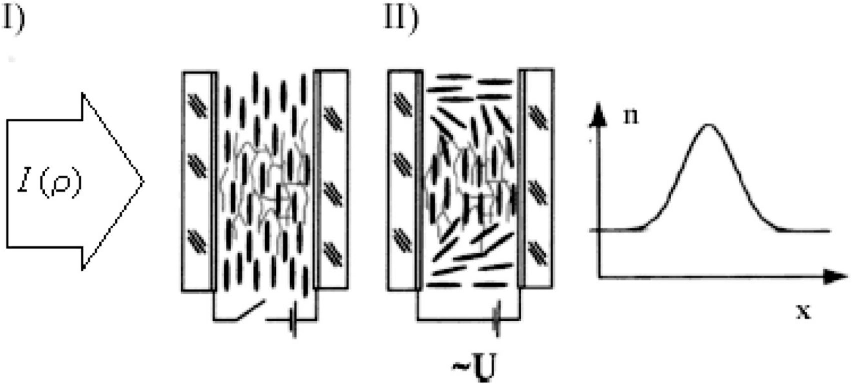

FIGURE 1 Director reorientation in NLC cell with network formation.

Section is devoted to the director reorientation under the action of externally applied voltage in the cell with polymer network. Finally in the third Section we study electro-optical properties of the lens, in particular focal length dependence on the applied voltage and upon the regime of photopolymerization.

2. POLYMER NETWORK FORMATION

The aim of this section is to get some insights onto spatial distribution of polymer concentration and its dependence on the intensity of curing UV light. We shall use classical scheme of radical photopolymerization [12,13]:

$$\begin{gathered} I + h\nu \to I^*, \quad I^* \to 2\dot{R}, \quad \dot{R} + M \to \dot{P}_1, \quad \dot{P}_n + M \to \dot{P}_{n+1}, \\ \dot{R} + \dot{R} \to I, \quad \dot{R} + \dot{P}_n \to P_n, \quad \dot{P}_{n_1} + \dot{P}_{n_2} \to P_{n_1+n_2}. \end{gathered} \tag{1}$$

Where I and I^* denote the photo initiation complex in ground and excited states, respectively; R is the radical, M is the monomer and P is the polymer.

Let N_m is monomer concentration, N_i is the concentration of photoinitiator, N_i^* is the concentration of excited molecules of photoinitiator, N_r is the concentration of radicals, $\widetilde{N}_p^n$ is the concentration of radicals with the length of n monomers, N_p^n is the concentration of polymer with the length n, then kinetic equations describing the photopolymerization reaction are:

photo-excitation of initiator

$$\begin{aligned} \frac{\partial N_i}{\partial t} &= -\zeta I(\rho) N_i + k N_r^2 \\ \frac{\partial N_i^*}{\partial t} &= \zeta I(\rho) N_i - \frac{N_i^*}{\tau_i} \end{aligned} \tag{2}$$

production of radical molecule

$$\frac{\partial N_r}{\partial t} = 2\frac{N_i^*}{\tau_i} - 2kN_r^2 - k_1 N_m N_r - k_2 N_r \sum_{n=1}^{\infty} \widetilde{N}_p^n \tag{3}$$

change of monomer concentration

$$\frac{\partial N_m}{\partial t} = -k_1 N_m N_r - k_3 N_m \sum_{n=1}^{\infty} \widetilde{N}_p^n \tag{4}$$

radical chain growth

$$\begin{aligned}\frac{\partial \widetilde{N}_p^1}{\partial t} &= k_1 N_m N_r - k_3 \widetilde{N}_p^1 N_m - k_4 \widetilde{N}_p^1 \sum_{n_1=1}^{\infty} \widetilde{N}_p^{n_1} - k_5 \widetilde{N}_p^1 N_r, \\ \frac{\partial \widetilde{N}_p^{n+1}}{\partial t} &= k_3 \widetilde{N}_p^n N_m - k_3 \widetilde{N}_p^{n+1} N_m - k_4 \widetilde{N}_p^{n+1} \sum_{n_1=1}^{\infty} \widetilde{N}_p^{n_1} - k_5 \widetilde{N}_p^{n+1} N_r\end{aligned} \tag{5}$$

reaction termination with final product of length n polymer chain

$$\frac{\partial N_p^n}{\partial t} = k_2 \widetilde{N}_p^n N_r + k_4 \sum_{n_1+n_2=n}^{\prime} \widetilde{N}_p^{n_1} \widetilde{N}_p^{n_2} \tag{6}$$

here the prime sign is introduced to avoid double counting.

In the above equations we shall suppose that the constants of chain prolongation and termination are independent on the chain length [12], $k_1 = k_3, k_2 = k_4 = k_5 = k$. Usually both the build-up time of the steady-state concentration of initial radicals and the lifetime of initiator molecules in the intermediate state are typically much smaller than the characteristic time of a polymerization process. Therefore supposing that the radical concentration can be constant during the photo-polymerization process [12] we get for steady state of the process

$$\frac{\partial N_r}{\partial t} = 0 = 2\zeta I(\rho) N_i - k N_r^2 - k_1 N_m N_r - k N_r \Sigma \tag{7}$$

where $\Sigma = \sum_{n_1=1}^{\infty} \widetilde{N}_p^{n_1} + N_r$ is the sum of all radical concentrations. The first equation of the system (5) now reads

$$\frac{\partial \widetilde{N}_p^1}{\partial t} = 0 = k_1 N_m N_r - \widetilde{N}_p^1 (k_1 N_m + k\Sigma) \tag{8}$$

Summation over all n in the second equation of the system (5) gives

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{\partial \widetilde{N}_p^{n+1}}{\partial t} = 0 = k_1 \widetilde{N}_p^1 N_m - k(\Sigma - N_r)\left(\Sigma - N_p^1 - N_r\right) \\ - k N_r \left(\Sigma - N_p^1 - N_r\right)\end{aligned} \tag{9}$$

On adding Eqs. (7)–(9) one obtains

$$\zeta I(\rho) N_i = k_2 \Sigma^2 \tag{10}$$

Including monomer diffusion into consideration we get the kinetic equation for monomer concentration

$$\frac{\partial N_m}{\partial t} = D\Delta N_m - k_1 N_m \sqrt{\zeta I(\rho) N_i / k} \tag{11}$$

From the second equation of the system (5) we can write relation between $\widetilde{N}_p^{n+1}$ and $\widetilde{N}_p^{n}$

$$\widetilde{N}_p^{n+1} = \widetilde{N}_p^{n} \frac{k_1 N_m}{k_1 N_m + k\Sigma} \approx \widetilde{N}_p^{n} \left(1 - \frac{k\Sigma}{k_1 N_m}\right) \equiv p\widetilde{N}_p^{n} \tag{12}$$

where we accounted that monomer concentration is much higher then total concentration of radicals, $\Sigma/N_m \ll 1$.

Also Eq. (8) gives us $\widetilde{N}_p^{1} = pN_r$ on the other hand by the definition we have

$$\Sigma = \sum_{n_1=1}^{\infty} \widetilde{N}_p^{n_1} + N_r = N_r \left(\sum_{0}^{\infty} p^n\right) = \frac{N_r}{1-p} \tag{13}$$

So finally we get for radical concentration $\widetilde{N}_p^{n} = \Sigma(1-p)p^n$.

Now to find the concentration of polymer with length n one can use the Eq. (6)

$$\begin{aligned}\frac{\partial N_p^n}{\partial t} &= k\Sigma^2(1-p)^2 p^n + k\Sigma^2(1-p)^2 \sum_{n_1+n_2=n}' p^n \\ &= k\Sigma^2(1-p)^2(n/2+1)p^n \end{aligned} \tag{14}$$

Total concentration of polymerized molecules is given by the summation of N_p^n multiplied by the chain length n

$$\frac{\partial N_p}{\partial t} = k\Sigma^2(1-p)^2 \sum_{n=n_0}^{\infty} n\left(\frac{n}{2}+1\right)p^n \tag{15}$$

here n_0 defines the smallest chain length that will force LC director to rotate back to the initial planar orientation. Supposing n_0 to be high enough one can find the sum

$$\frac{\partial N_p}{\partial t} \approx \frac{1}{2}\frac{k\Sigma^2 p^{n_0}}{1-p}\left(n_0^2(1-p)^2 + 2n_0(1-p) + p(3-p)\right) \tag{16}$$

One can distinguish two boundary cases:
$n_0(1-p) \gg 1, (1-p) \ll 1$ and $n_0(1-p) \ll 1$. In the first one we have approximately

$$\frac{\partial N_p}{\partial t} \approx \frac{1}{2} k n_0^2 \Sigma^2 \frac{k\Sigma}{k_1 N_m}\left(1 - \frac{k\Sigma}{k_1 N_m}\right)^{n_0} \sim I^{3/2}(\rho)/N_m \tag{17}$$

and in the second boundary case

$$\frac{\partial N_p}{\partial t} \approx k\Sigma^2 \left(\frac{k\Sigma}{k_1 N_m}\right)^{-1} \left(1 - \frac{k\Sigma}{k_1 N_m}\right)^{n_0+1} \sim I^{1/2}(\rho) N_m \tag{18}$$

Monomers from outside part of the illuminated spot will diffuse through the polymerization region with the characteristic time of $\tau_D \sim R^2/D$. Taking the monomer diffusion constant D to be $10^{-10}\ \mathrm{m}^2/\mathrm{s}$ [14], $R \sim 10^{-3}$ m [10] one finds $\tau_D \sim 10^4$ s. In what follows we suppose that characteristic time of photopolymerization process is much longer than monomer diffusion. Therefore during the photopolymerization reaction we can suppose the concentration of monomers in Eqs. (17), (18) to be constant, and hence (17) and (18) give us spatial distribution of polymerized network. Slow monomer diffusion case we shall study elsewhere. From these equations it also follows that polymer concentration is linear function of polymerization time. Both cases (17) and (18) can be described now by one equation: $N_p(\rho) = N_p(0)t\exp(-\gamma\rho^2)$ with different parameters $N_p(0)$ and $\gamma = 1.5\alpha$ or $\gamma = 0.5\alpha$. Experimental data of paper (Fig. 2 in [10]) confirms the first case.

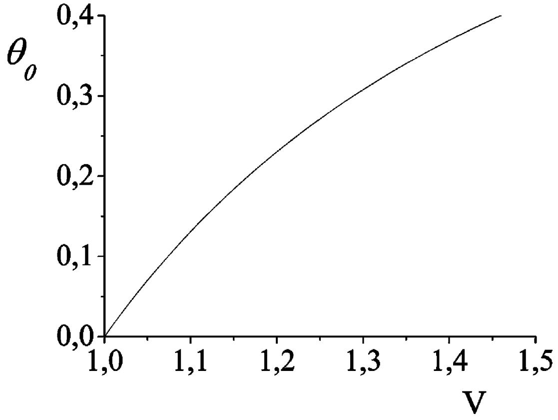

FIGURE 2 Parameter θ_0 in (26) as a function of $V = U/U_0^{th}$.

3. SPLAY GEOMETRY DIRECTOR REORIENTATION IN POLYMER STABILIZED LC UNDER THE EXTERNAL ELECTRIC FIELD

Consider a nematic LC cell with the thickness L and uniform planar boundary conditions with strong anchoring. At the cell surfaces director is parallel to the OX axis. We apply electric field along the OZ axis. Now from the symmetry reason the director field is given by

$$\mathrm{n} = (\cos\theta(\rho,z), 0, \sin\theta(\rho,z)) \tag{19}$$

The thermodynamic functional is given by

$$F = \frac{1}{2}\left[K_{11}\int(\mathrm{div}\mathbf{n})^2 dV + K_{22}\int(\mathbf{n}\cdot\mathrm{curl}\mathbf{n})^2 dV + K_{33}\int(\mathbf{n}\times\mathrm{curl}\mathbf{n})^2 dV\right] - \frac{W}{2}\int N_p(\mathbf{r})(\mathbf{n}\cdot\mathbf{e})^2 dV - \frac{1}{2}\int(\mathbf{D}\cdot\mathbf{E})^2 dV \tag{20}$$

where the first three terms are usual elastic contribution to the LC total free energy, the forth term describes torque which is due to presence of polymer network formed by UV illumination with the beam of Gaussian shape, N_p is the polymer concentration (depending on the regime of curing it depends on intensity according to (17) or (18)), $N(\rho) = \overline{N}_p(0)\exp(-\gamma\rho^2)$ (relation between γ and α depends on the regime of polymerization, i.e., Eqs. (17) or (18)), $\mathbf{D}$ is the electric displacement vector, $\mathbf{D} = \hat{\varepsilon}\mathbf{E}, \varepsilon_{ij} = \varepsilon_\perp\delta_{ij} + (\varepsilon_{||} - \varepsilon_\perp)n_i n_j$.

To get the magnitude of parameter W one may measure the Freedericksz transition threshold shift in pure cell and illuminated with plane UV beam. Below we suppose that there are no free charges in the LC cell, therefore the displacement vector satisfies the equation

$$\mathrm{div}\mathbf{D} = 0 \tag{21}$$

Now we account that characteristic length of electric field inhomogeneity in z direction is L and in XOY plane it is R. $R >> L$, therefore we neglect $\partial/\partial\rho$ derivatives in comparison with $\partial/\partial z$ derivatives and get respectively $D_z = \mathrm{const}$.

Since $\mathrm{curl}\mathbf{E} = 0$ and taking into account that electric field inside the cell depend on z coordinate only, we get $E_x = \mathrm{const}$ and $E_y = \mathrm{const}$. At the cell walls $E_x = E_y = 0$, therefore we have $\mathbf{E} = (0, 0, E(z))$. As the result $D_z = \varepsilon_{zz}E_z = (\varepsilon_\perp + \varepsilon_a\sin^2\theta(z))E_z = \varepsilon_\perp E_0 = \mathrm{const}$ and, consequently, the voltage U across the nematic cell

$$U = \int_0^L E dz = D_z\int_0^L \left(\varepsilon_\perp + \varepsilon_a\sin^2\theta(z,\rho)\right)^{-1} dz \tag{22}$$

The thermodynamic functional per unit area takes the form

$$F = \frac{1}{2}K\int\left[(\theta'_z)^2+\left(\theta'_\rho\right)^2\right]dV - \frac{W}{2}\int N_p(\mathbf{r})\cos^2\theta\,dV - \frac{1}{2}\int D_z^2\left(\varepsilon_\perp+\varepsilon_a\sin^2\theta\right)^{-1}dV \tag{23}$$

Here we consider approximation of one elastic constant $K_1 = K_2 = K_3 = K$. Minimisation of this functional should be done subject to constraint Eq. (22). We re-write the free energy functional in terms of voltage instead of dielectric displacement

$$\begin{aligned} F/2\pi = {} & \frac{1}{2}K\int_0^\infty \rho d\rho\int_0^L\left[(\theta'_z)^2+\left(\theta'_\rho\right)^2\right]dz \\ & -\frac{W}{2}\int_0^\infty \rho d\rho\int_0^L N_p(\rho)\cos^2\theta(z,\rho)\,dz \\ & -\frac{U^2}{2}\int_0^\infty \rho d\rho\left[\int_0^L\left(\varepsilon_\perp+\varepsilon_a\sin^2\theta(z,\rho)\right)^{-1}dz\right]^{-1} \end{aligned} \tag{24}$$

For small values of the director deviation from its initial value functional (24) takes the form

$$\begin{aligned} F/2\pi = {} & \frac{1}{2}K\int_0^\infty \rho d\rho\int_0^L\left[(\theta'_z)^2+\left(\theta'_\rho\right)^2\right]dz \\ & -\frac{W}{2}\int_0^\infty \rho d\rho\int_0^L N_p(\rho)\left(1-\theta^2(z,\rho)+\frac{1}{3}\theta^4(z,\rho)\right)dz \\ & -\frac{U^2\varepsilon_\perp}{2}\int_0^\infty \rho d\rho\left[\int_0^L\left(1-\frac{\varepsilon_a}{\varepsilon_\perp}\theta^2(z,\rho)+\theta^4(z,\rho)\right.\right. \\ & \qquad \left.\left.\times\left(\frac{\varepsilon_a}{3\varepsilon_\perp}+\left(\frac{\varepsilon_a}{\varepsilon_\perp}\right)^2\right)\right)dz\right]^{-1} \end{aligned} \tag{25}$$

First we consider the voltages above the Freedericksz transition voltage in pure LC, but less than transition voltage in the centre of polymerizing beam spot. It is plausible to seek director profile in the form

$$\theta = \left\{\theta_0\left[1-\exp\left(-\beta\rho^2\right)\right]\right\}^{1/2}\sin\frac{\pi z}{L} \tag{26}$$

Substituting (26) in (25) and performing integration over z we get the function to be minimized

$$\begin{aligned}\Phi(\theta_0,\beta) = \frac{KL}{4}\int_0^\infty \frac{\pi^2\theta_0}{L^2}\{1-\exp(-\beta\rho^2)\}\rho d\rho \\ -\frac{\xi K}{2L^2}\int_0^\infty \exp(-\gamma\rho^2)\left[1-\frac{1}{2}\theta_0(1-\exp(-\beta\rho^2))\right. \\ \left.+\frac{1}{8}\theta_0^2(1-2\exp(-\beta\rho^2)+\exp(-2\beta\rho^2))\right]\rho d\rho \\ -\frac{U^2\varepsilon_\perp}{2L}\int_0^\infty \left[1+\frac{\varepsilon_a\theta_0}{2\varepsilon_\perp}(1-\exp(-\beta\rho^2))\right. \\ \left.-\frac{\varepsilon_a}{8\varepsilon_\perp}\left(1+\left(\frac{\varepsilon_a}{\varepsilon_\perp}\right)\right)\theta_0^2(1-2\exp(-\beta\rho^2)+\exp(-2\beta\rho^2))\right]\rho d\rho\end{aligned} \tag{27}$$

where we neglected the derivative $\partial/\partial\rho$ in comparison with the derivative $\partial/\partial z$ because characteristic length of director inhomogeneity in z-direction (cell thickness) is much less than the characteristic size of the director inhomogeneity in ρ (size of beam spot).

From the expression (27) it is easy to find Freedericksz transition voltage in LC cell with homogeneous distribution of polymer network:

$$\frac{U_{UV}^{th}}{U_0^{th}} = \sqrt{1+\frac{\xi}{\pi^2}} \tag{28}$$

To estimate the parameter ξ we used the experimental data ([11], Fig. 3). Freedericksz transition in pure LC occurs at 0.98 V, for polymer stabilized cell black area in the centre of testing beam spot disappears at 1.45 V, therefore $U_{UV}^{th}/U_0^{th} \approx 1.45/0.98 = 1.48$, that corresponds to parameter $\xi \approx 11$.

The necessary conditions of minimum $\partial\Phi/\partial\theta_0 = 0$, $\partial\Phi/\partial\beta = 0$ lead to the following system of equations for parameters θ_0 and β:

$$\theta_0 = 2\frac{1-V^{-2}}{\left(1+\frac{\varepsilon_a}{\varepsilon_\perp}\right)},$$

$$\begin{aligned}\frac{\xi x^2}{\pi^2}\left(\frac{1}{(x+1)^2}+\frac{\theta_0}{4}\left(\frac{1}{(x+1)^2}-\frac{1}{(2x+1)^2}\right)\right) \\ -\frac{3\theta_0 V^2}{2}\left(1-\frac{1}{4}\left(1+\frac{\varepsilon_a}{\varepsilon_\perp}\right)\right)+1=0\end{aligned} \tag{29}$$

where $x = \beta/\gamma$.

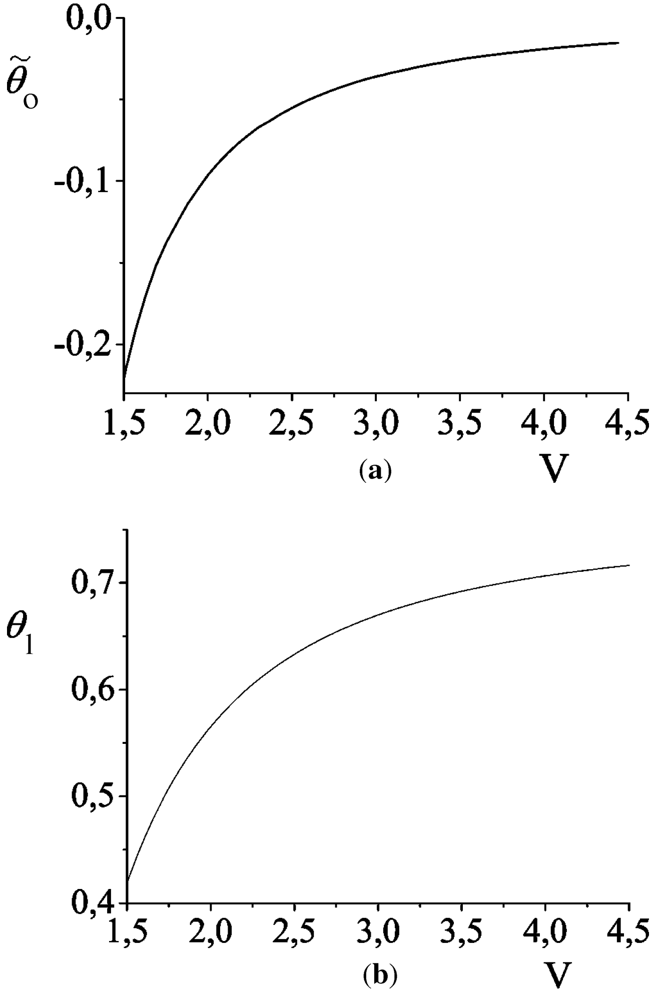

FIGURE 3 Parameters $\tilde{\theta}_0$ (a) and θ_1 (b) in (30) as a function of $V = U/U_0^{th}$.

At voltages above the Freedericksz transition in the centre of polymerizing beam spot we shall seek minimum of the functional (25) delivered by the function $\theta(z,\rho)$ of the form

$$\theta(\rho,z) = \left(\tilde{\theta}_0 \exp\left(-\beta\rho^2\right) + \theta_1\right)^{1/2} \sin\frac{\pi z}{L} \qquad (30)$$

Making similar calculations for probe function parameters $\tilde{\theta}_0(U,\gamma)$, $\theta_1(U)$ and $\beta(U,\gamma)$ we get the system of equations

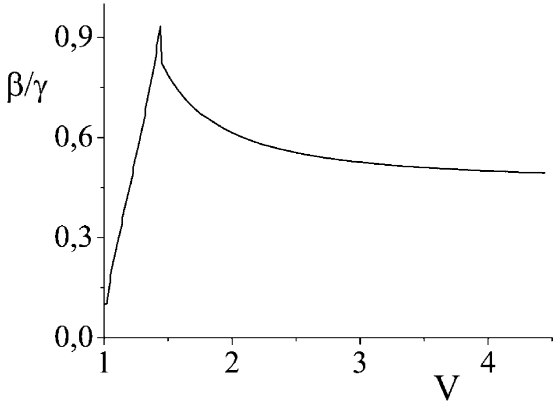

FIGURE 4 Parameter β in (26), (30) as a function of $V = U/U_0^{th}$.

$$\theta_1 = 2\frac{1 - V^{-2}}{\left(1 + \frac{\varepsilon_a}{\varepsilon_\perp}\right)},$$

$$\begin{aligned}
\tilde{\theta}_0 &= \xi\frac{x}{(x+1)\pi^2}\frac{1 - \frac{(1-V^{-2})}{1+\left(\frac{\varepsilon_a}{\varepsilon_\perp}\right)}}{\xi\frac{x}{2(2x+1)\pi^2} - \left(1 + \frac{\varepsilon_a}{\varepsilon_\perp}\right)\frac{V^2}{4}}, \\
&\frac{\xi}{\pi^2}\left(\frac{x^2}{(x+1)^2} + \frac{1}{4}\left(\theta_1\frac{x^2}{(x+1)^2} + \tilde{\theta}_0\frac{x^2}{(2x+1)^2}\right)\right) \\
&- V^2\left(1 - \frac{1}{4}\left(1 + \frac{\varepsilon_a}{\varepsilon_\perp}\right)\left(2\theta_1 + \frac{\tilde{\theta}_0}{2}\right)\right) + 1 = 0
\end{aligned} \tag{31}$$

The systems of Eqs. (29) and (31) can easily be solved. Figures 2, 3 and 4 show the dependences of parameters $\tilde{\theta}_0$, θ_1 and β/γ on the applied voltage at $\xi = 11$, dielectric constant $\varepsilon_{||} = 19, \varepsilon_\perp = 5.2$ (LC E7 [15]), cell thickness $L = 10\,\text{mkm}$, $\gamma = 3\,\text{mm}^{-2}$.

4. LIGHT PROPAGATION THROUGH POLYMER STABILIZED LC SUBJECT TO EXTERNALLY APPLIED VOLTAGE

In the previous section we found spatial profile of LC director subject to externally applied voltage. Refractive index for extraordinary beam propagating in the LC cell is given by

$$n(\psi) = \frac{\sqrt{\tilde{\varepsilon}_{||}\tilde{\varepsilon}_\perp}}{\left(\tilde{\varepsilon}_{||}\cos^2\psi + \tilde{\varepsilon}_\perp\sin^2\psi\right)^{1/2}} \tag{32}$$

where ψ is the angle between director (optical axis) and light beam wave vector, $\hat{\tilde{\varepsilon}}$ is the dielectric tensor at optical frequency, in our case $\psi = \pi/2 - \theta$. For small director deviation from the planar alignment ($\theta(x,z) \ll 1$) refractive index for e-wave will be

$$n(z,\rho) = \sqrt{\tilde{\varepsilon}_{||}} - \frac{1}{2}\frac{\sqrt{\tilde{\varepsilon}_{||}}\tilde{\varepsilon}_a}{\tilde{\varepsilon}_\perp}\theta^2(\rho,z) \tag{33}$$

We can say now that on switching on the electric field the lenslike gradient refractive index profile is created in the cell. This gradient is due to the spatially distributed polymer network obtained by photopolymerization using a spatially nonuniform laser beam. Focusing property of the obtained lens is determined by the phase retardation dependence on the distance from the beam centre. The focal length of this lens can be estimated using Fresnel approximation [16,17]. Below we shall treat LC cell illuminated with testing beam like a thin phase plate. Then phase retardation for the test light beam passing through the cell will depend on the distance from the centre of UV Gaussian beam that created polymer network

$$\phi(\rho,U,w,\gamma) = \frac{2\pi}{\lambda}\sqrt{\tilde{\varepsilon}_{||}}L - \frac{\pi}{\lambda}\frac{\sqrt{\tilde{\varepsilon}_{||}}\tilde{\varepsilon}_a}{\tilde{\varepsilon}_\perp}\int_0^L \theta^2(\rho,z)dz \tag{34}$$

Using solution for $\theta(\rho,z)$ (30) it is easy to find

$$\phi(\rho,U,w,\gamma) = \frac{2\pi L}{\lambda}\sqrt{\tilde{\varepsilon}_{||}} - \frac{\pi L\varepsilon_a\sqrt{\tilde{\varepsilon}_{||}}}{2\lambda\tilde{\varepsilon}_\perp}\left(\tilde{\theta}_0\exp\left(-\beta\rho^2\right) + \theta_1\right) \tag{35}$$

For small ρ we can expand the last expression (35) for phase retardation and get

$$\phi(\rho,U,w,\gamma) = \frac{2\pi L}{\lambda}\sqrt{\tilde{\varepsilon}_{||}} - \frac{\pi L\varepsilon_a\sqrt{\tilde{\varepsilon}_{||}}}{2\lambda\tilde{\varepsilon}_\perp}\left(\theta_1 + \tilde{\theta}_0\right) + \frac{\pi L\varepsilon_a\sqrt{\tilde{\varepsilon}_{||}}}{2\lambda\tilde{\varepsilon}_\perp}\tilde{\theta}_0\beta\rho^2 \tag{36}$$

The focal length of this lens in Fresnel approximation reads

$$f = -\frac{2\tilde{\varepsilon}_\perp}{\tilde{\varepsilon}_a L\sqrt{\tilde{\varepsilon}_{||}}\tilde{\theta}_0(U,\gamma)\beta(U,\gamma)} \tag{37}$$

On Figure 5 we show the focal length dependence on the applied voltage (LC E7 $\sqrt{\varepsilon_{||}} = n_e = 1.738$, $\sqrt{\varepsilon_\perp} = n_o = 1.518$ [18])

Being qualitatively the same, our theoretical results partially disagree with experimental data of [10,11]. We got focal length changes from 0.6 m to 16 m for voltages range 1.5–4.5 V, while the authors of [10,11] estimated focal length being 1.6–8 m. Their estimations are

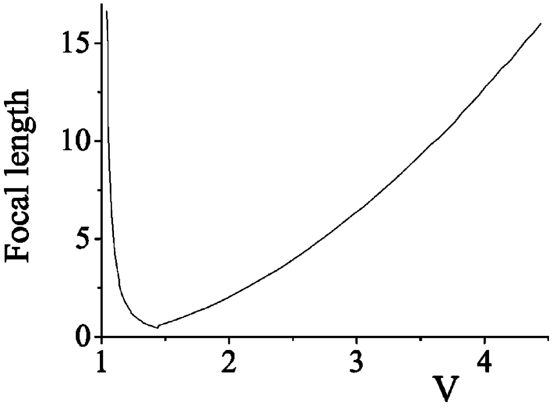

FIGURE 5 Focal length dependence on the applied voltage $V = U/U_0^{th}$.

based on the experimental data of phase retardation. In [10,11] it was supposed that lens radius does not change with the increase of voltage, that may be not the case. Therefore the authors might be underestimated the focal length. The other reason for discrepancy could be slow monomer diffusion that was neglected in our consideration.

5. CONCLUSION

Here we present theoretical model that describes the dependence of the gradient polymer stabilized liquid crystal lens focal length on applied voltage. The profile of the polymer network concentration has a Gaussian form. Minimizing the total free energy functional we find the director profile in the cell subject to externally applied electric field. Freedericksz transition threshold appears to be maximal in the centre of the illuminated spot and minimal in the nonilluminated regions. We suppose that polymer network creates additional torque onto LC director forcing it to preserve planar orientation despite applied voltage. The model qualitatively agrees with the experimental data. Obtained results can be applied to develop gradient polymer stabilized liquid crystal lenses that have no moving parts and allow the electro-optical zooming.

REFERENCES

[1] Kowel, S. T., Cleverly, D. S., & Kornreich, P. G. (1984). *Appl. Opt.*, *23*, 278.
[2] Nose, T. & Sato, S. (1989). *Liq. Cryst.*, *5*, 1425.
[3] Nose, T. & Sato, S. (1991). *Jpn. J. Jppl. Phys.*, *30*, L2110.

[4] Nose, T., Masuda, S., & Sato, S. (1992). *Jpn. J. Appl. Phys.*, *31*, 1643.
[5] Riza, N. A. & DeJule, M. (1994). *C. Opt. Lett.*, *19*, 1013.
[6] Naumov, F., Loktev, M. Yu., Guralnik, I. R., & Vdovin, G. (1998). *Opt. Lett.*, *23*, 992.
[7] Gvozdarev, A. & Nevskaya, G. E. (1999). *Mol. Cryst. Liq. Cryst.*, *329*, 693.
[8] Patel, J. S. & Rastani, K. (1991). *Opt. Lett.*, *16*, 532.
[9] Commander, L. G., Day, S. E., & Selviah, D. R. (2000). *Opt. Commun.*, *177*, 157.
[10] Presnyakov, V. V., Asatryan, K. E., Galstian, T. V., & Tork, A. (2002). *Opt. Express*, *10*, 865.
[11] Presnyakov, V. & Galstian, T. V. (2005). *J. of App. Phys.*, *97*, 103101.
[12] Atkins, P. W. (1987). *Physical Chemistry*, Oxford University Press: Oxford.
[13] Caputo, R., Sukhov, A. V., Tabiryan, N. V., Umeton, C., & Ushakov, R. F. (2001). *Chem. Phys.*, *271*, 323.
[14] Caputo, R., Sukhov, A. V., Tabiryan, N. V., Umeton, C. (1999). *Chem. Phys.*, *245*, 463.
[15] Drzaic, P. S. & Muller, A. (1989). *Liq. Cryst.*, *5*, 1467.
[16] Altshuler, G. B. & Inochkin, M. V. (1993). *Soviet Physics Uspehi*, *163*, 65.
[17] Goodman, J. W. (2002). *Introduction to Fourier Optics* (Second Edition), McGraw-Hill International Editions.
[18] Wang, M., Li, W., Zou, Y., & Pan, C. (1997). *J. Phys. D*, *30*, 1815